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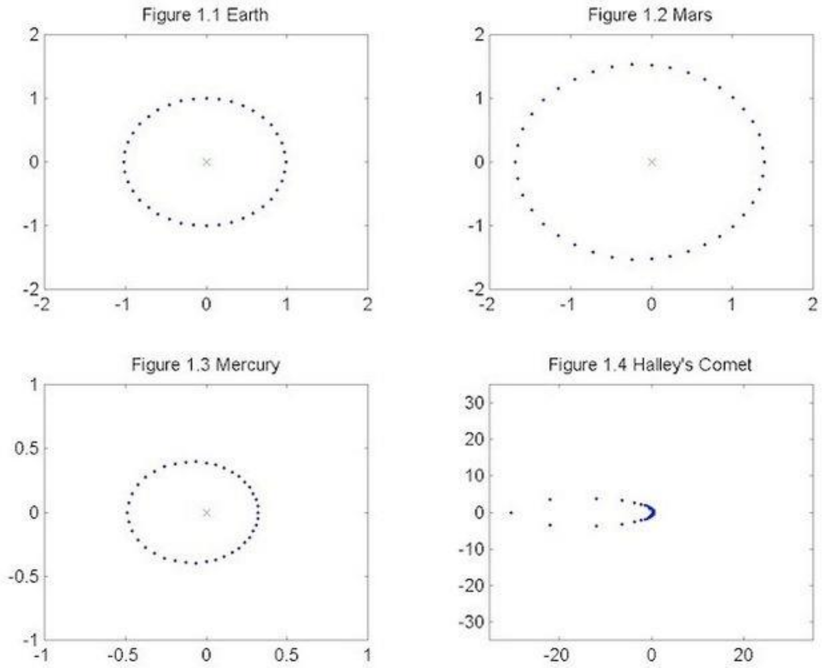

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Gravitational Choreography

By Mike Frost

Let me start, as I did in 1987, with the Czech astronomer Johannes Kepler. Kepler was a student and collaborator of the Danish astronomer Tycho Brahe, who painstakingly compiled the largest and most accurate catalogue of star positions of the pre-telescopic era. With the aid of Tycho's catalogue, his own observations, and a tremendous amount of tedious computation, Kepler was able to announce three laws of planetary motion. The first two, dating from 1609, were as follows:

1/ A planet moves round the Sun in an ellipse, with the Sun at one focus

2/ The line joining a planet to the Sun sweeps out equal areas in equal times

Ten years later, in 1619, Kepler announced a third law.

3/ The cube of the semi-major axis of the ellipse of a planet's orbit is proportional to the square of its orbital period.

Don't worry about the details of these just yet! The single most important detail to Kepler's Laws is that the planets do not orbit the Sun in circular orbits. This contradicted the most accurate previous model of the Solar System, proposed by Nicolaus Copernicus in 1543. Copernicus had correctly deduced that the planets orbited the Sun rather than the Earth; however, with less accurate data, he had stated that the orbits were perfect circles.

It is easy to understand why Copernicus arrived at that conclusion. If you look at an accurate diagram of the orbits of most of the planets, most of them appear very nearly circular. Of the planets known to Copernicus and Kepler, only Mercury and Mars are clearly non-circular at a quick glance (Pluto is clearly non-circular, as well, but it wasn't known to Kepler and arguably it's not a planet anyway). The most convenient measure of the non-circularity of a planet's orbit is the "eccentricity" of the ellipse. An eccentricity of 0 gives a circle; as the eccentricity rises the orbit becomes more and more squashed. As the eccentricity approaches 1 the circle becomes very squashed indeed, taking on a cigar shape. Technically an eccentricity of 1 is a parabola, a curve that never closes up at all.

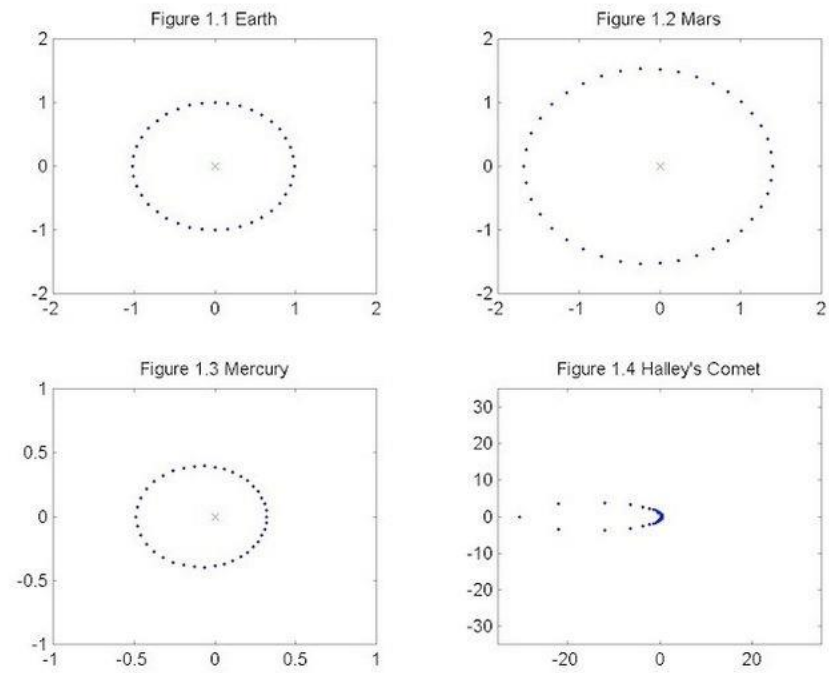


Figure 1 — Orbits of various objects in the Solar System: Earth, Mars, Mercury and Halley's Comet.

The Sun is at the origin (0,0) of each graph. The units are Astronomical Units or A.U. (1.0 A.U. = the mean distance of the Earth from the Sun). The distances for Halley's comet are approximate. Each point of the orbit is plotted at an equal time interval. Kepler's second law tells us that the distance between points is smallest when the object is closest to the Sun, and greatest when it is furthest. This is clearest to see with Halley's comet.

Figure 1 above, shows the orbit of Earth, which has eccentricity 0.0167; Mars, with eccentricity 0.0934; Mercury, with eccentricity 0.2056, and Halley's comet, with eccentricity 0.967. I'm getting a little ahead of the story by including the comet, as Halley didn't come on the scene until the end of the seventeenth century — nevertheless the comet which bears his name also obeys Kepler's Laws, with a much more squashed orbit than either Earth, Mars or Mercury. You'll notice that I've represented the orbits as a series of dots, marking out equal times. Earth's orbit marks out the dots almost at equal separations, but Halley's comet behaves completely differently, speeding up drastically as it approaches and sweeps round the Sun, and then slowing down again as it heads back out into the outer solar system. In fact both Earth and Halley's comet are demonstrating Kepler's second Law — the line from the object to the Sun sweeps out equal areas in equal times.

Kepler's Laws were a huge step forward to telling us what was going on the in Solar System, but they failed to explain why. The explanation of why was provided in 1687, in perhaps the most important scientific work ever published — Isaac Newton's *"Principia Mathematica"*.

Newton's Revolutions

Now is not the time to tell the story, fascinating though it is, of how Isaac Newton's masterpiece had to be prised from its reluctant author. Suffice it to say that without the efforts of Edmund Halley, of comet fame, we might never have known how deeply Isaac Newton had come to understand the workings of the Solar System. Principia Mathematica embodies not one revolution in scientific thinking but three. First of all were Newton's Laws of motion:

1/ A body not acted on by any force moves in a straight line with constant velocity.

2/ A body acted on by a force undergoes an acceleration such that force equals mass times acceleration.

3/ A body acted on by a force exerts an equal and opposite force.

These laws continue the development of the theory of motion begun by Galileo in the early part of the seventeenth century. They tell us how everything — planets, comets, apples, pumas — moves.

The second revolution was the development of the mathematical theory of calculus, or fluxions, as Newton called it. This comprised a set of radical new mathematical techniques that enabled Newton to analyse problems of motion to a depth never dreamt of previously. I ought to add two comments; first, that the calculus was also developed independently of Newton, by Gottfried Leibnitz in Germany; and second, that, having developed his theories, Newton rewrote all his proofs in Principia using more conventional geometrical arguments. Nonetheless, to produce Principia Mathematica, Newton invented much of what is now the A-level mathematics syllabus [3].

The third revolution, and the one that is most relevant to us here, was the theory of universal gravity. Newton's theory of gravity gives the force between two bodies of mass M and m, at distance r apart. The force acts along the line joining the two bodies and has size proportional to the inverse square of the distance between them

$$F = \frac{-GMm}{r^2}$$

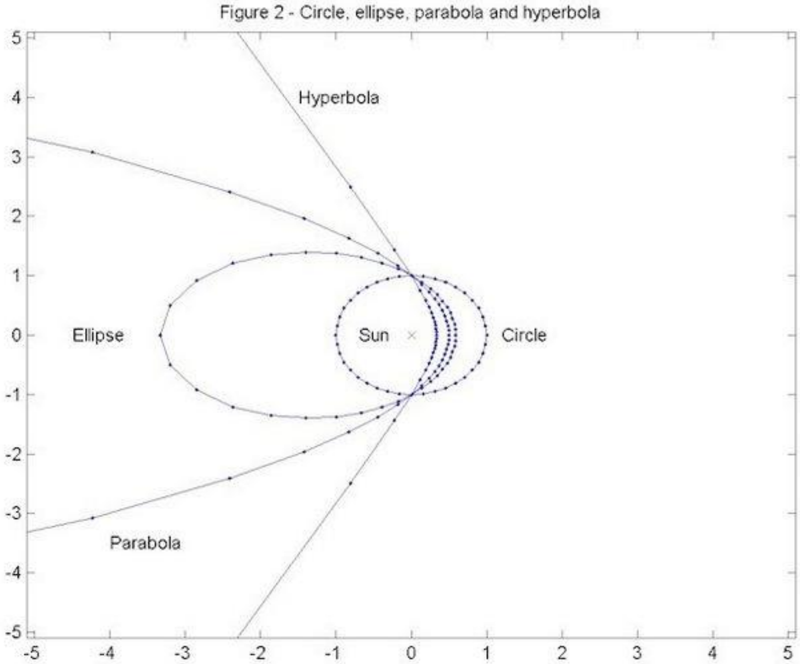
Newton brought these three revolutions together to show that, using calculus and the laws of motion, Kepler's laws could be deduced from the law of gravity. Moreover, Newton demonstrated that the theory of gravity held wherever it could be observed. The law of gravity that kept the Earth in its orbit around the Sun was the same law that predicted the fall of the apple, and the orbit of Jupiter's moons, and the motion of a projectile.

In fact, there were a few refinements to Kepler's laws. The first law, as stated, isn't strictly accurate. In fact both planet and Sun move in elliptical orbits around the centre of gravity of the two. One can forgive Kepler this error, because the Sun is much bigger than any of the planets, and so the centre of gravity for the Sun and any given planet lies within the surface of the Sun. For more equal binary pairs the mutual ellipses are more obvious, and lead to the "wobble" in the motion of stars from which the existence of planets outside the solar system can be deduced.

Newton's law also allows for another type of orbit — a hyperbolic orbit. In this case, an object approaches the Sun from infinity and then flies away to infinity again. Such orbits can be classified using eccentricities greater than 1.0, and they have been observed for comets arriving from the far reaches of the solar system.

With the addition of hyperbolic orbits, the classification of all possible orbits of one body around another is complete. If one body is orbiting another, it does so as either an ellipse, which is an orbit that closes up, or a hyperbola, which is an orbit that doesn't close up. The eccentricity of the orbit defines how elliptical or hyperbolic the orbit is (there are special extreme cases of eccentricity zero, a circle, and eccentricity one, a parabola, which are not observed in practice). Figure 2 shows these different types of orbits. In particular, there is only one type of periodic (that is to say, repeating) orbit — namely the ellipse. In a stable system consisting of two bodies, the only behaviour permitted by Newton's law of gravity is Kepler's first law — the two objects will orbit around their centre of gravity in ellipses.

Figure 2 - Circle, ellipse, parabola and hyperbola



But there are more than two objects in the universe. It is natural to ask what solutions there are to Newton's equations for more than two bodies — in particular what stable orbits three bodies will take up. And here there is a surprise. Newton, who completely solved the "two-body" problem by outlining all possible two-body orbits, was unable to find even one solution to the "three-body" problem. Not one stable periodic solution for three bodies orbiting round each other. And it was nearly eighty years before anybody else did any better.

It is possible these days to run computer simulations of gravity, starting from a chosen configuration and seeing how the system evolves, (for example `http://www.arachnophile.com/gravitation`). It is very instructive to do this. Go ahead, try it! I'll wait. . .

Multi-body systems in practice

Start with any three objects with randomly chosen masses in random locations and set them off. What usually happens is chaotic motion, non-repeating, until the smallest of the three bodies gets too close to one of the other two. The smallest object then suddenly picks up speed and is flung away from the other two. The two larger objects usually then settle down to a stable binary orbit. Another occasional outcome is a collision or merger between two of the three objects, leaving behind just two remaining objects in a stable binary orbit.

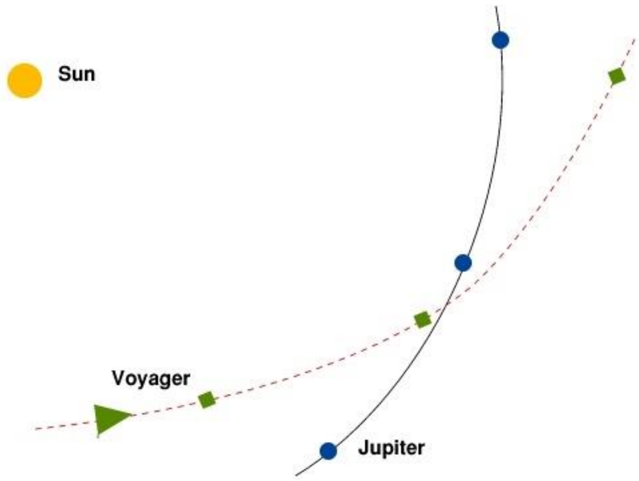


Figure 3 — The slingshot effect. Voyager picks up speed passing Jupiter. Voyager moves from one elliptical orbit to a second elliptical orbit at a greater speed, relative to Jupiter, Voyager is on a hyperbolic orbit as it passes the planet.

This sudden change of speed on a close approach is called the slingshot effect, as shown in Figure 3. The space agencies use the slingshot effect to speed up their interplanetary probes. Voyager 2, for example, got a slingshot from each of Jupiter, Saturn, Uranus and Neptune on its way out of the solar system [4].

So how does nature solve the three-body problem? How has the solar system evolved to stability when it contains more than two bodies? The answer is that the solar system has become a hierarchy of binary orbits. Essentially, every body in the solar system orbits the nearest "big object". For example, a moon of Jupiter such as Ganymede orbits Jupiter because that is the nearest big object — although the Sun is considerably more massive, it is much further away from Ganymede than Jupiter is and the gravitational force exerted on Ganymede by the Sun is much less than the force exerted by Jupiter. Each object in the solar system has, crudely speaking, a "zone of attraction" within which it can hold a retinue of satellites. At the boundary of the zone of attraction the gravitational hold is tenuous, and so satellites may be gained or lost. Jupiter's outer satellites, for example, are thought to be captured asteroids. Any object orbiting close to the boundary will probably not stay there for long.[5]

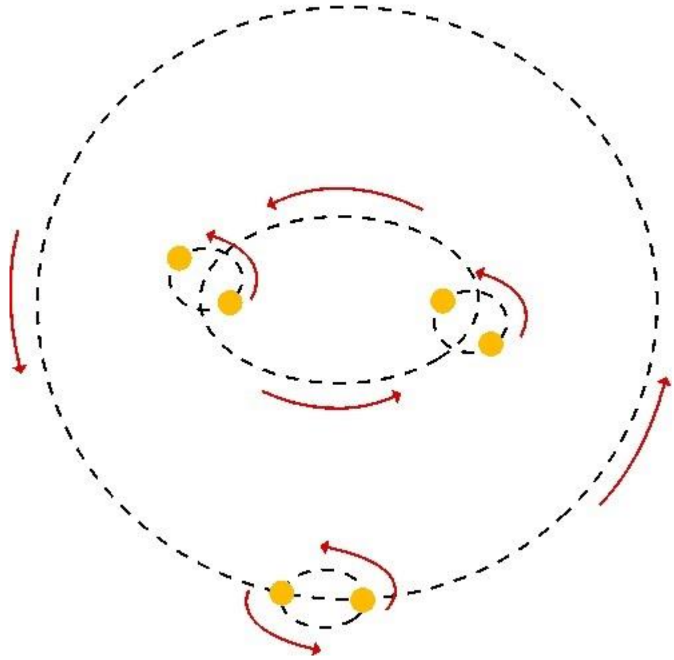


Figure 4 — Castor, a six star system consisting of three binary pairs

In part this scheme applies to the solar system because the objects within it also form a hierarchy. There is one large Sun, containing virtually all the mass in the solar system, with a number of planets orbiting it. The larger planets have substantial retinues of satellites, the smaller planets (and occasionally asteroids) may have one or two satellites. But what happens in a system where there is more than one "top dog"? To find out, we have to look at multiple star systems elsewhere in our own galaxy. One example is Castor, one of the two bright stars in Gemini. It turns out that Castor is not one star at all, but, remarkably, six! And how do these stars orbit each other? They orbit as a series of binary pairs. One tight binary pair orbits another tight binary pair, with yet another binary pair orbiting around the two pairs (see Figure 4 above). The six stars have found a stable set of orbits that consist of five binary systems, each obeying Kepler's first law. No radically new orbit has emerged.

Euler Orbits

The next step in predicting new orbital motions was taken by the prolific Swiss mathematician Leonhard Euler, in 1765. He envisaged a number of rather artificial constructions. For example, Figure 5 shows a large mass like the Sun being orbited by two identical smaller masses placed symmetrically either side of the central mass. What forces are felt by each of the three bodies? The central mass is tugged in opposite directions by each of the outer two masses, and consequently feels no residual force at all. Each of the two outer masses feels the gravitational force of both the central mass and the other smaller mass on the far side; however each of the two pulls on it is in the same direction.   The force each of the outer bodies feels is the same as if the other two bodies were replaced by just one, of greater mass, at the centre of the system.

Figure 5 — Euler's "Counter-Earth" configuration. Two identical masses m in orbit around mass M in identical orbits 180° apart

It is therefore possible for the two smaller masses to orbit, 180 degrees apart, in a circle around the centre mass. The configuration is reminiscent of the ancient concept of the counter-Earth, the planet that orbits, unknown to us, on the far side of the Sun. However, as we shall see, this configuration is not stable.

Euler discovered a second configuration that is worth considering in more detail. Suppose that you have three identical masses on the vertices of an equilateral triangle; or if you prefer, spaced one hundred and twenty degrees apart on a circle (it amounts to the same thing). The centre of mass of this system is at the centre of the circle. Where does the gravitational force on each particle pull it? Each particle is attracted equally towards the other two (forces which act along the sides of the triangle) but the sum of these two forces, because of the symmetry of the system, is a net force towards the centre of the circle — each mass is attracted to the centre of gravity. If the masses are simultaneously released from rest they will move smoothly to the centre of the circle and collide.

Figure 6 — Euler's triangular configuration. Three identical masses moving at the same speed, placed at 120° intervals around a circle. If the speed is to low (Left), the bodies move inwards, if to high (Right) the bodies move apart.

Only if the speed is just right (Centre) will the configuration persist.

Now consider the same system, but with the three masses spinning around the circle at constant speed. Now they will have an acceleration towards the centre of the circle due to their angular speed. Euler realised that, if the speed of rotation was just right, the acceleration would just match that required to satisfy the gravitational force, and the circular orbit for the three bodies would be stable. If the bodies are moving slower than the critical speed, they spiral in towards the centre of the circle; faster than the critical speed, and they fly away to infinity. But at the critical speed you have a new type of stable orbit for three bodies — three identical masses spinning round in a circle as an equilateral triangle (Figure 6 above).

Figure 7 — 4-body, 5-body, 6-body configurations

Euler also realised that you could use the same argument to generate stable systems for four, five, six or indeed any number of bodies — four identical masses orbiting in a square, five as a regular pentagon, and so on (Figure 7 above). But, unfortunately, he knew full well that none of these systems would ever be observed in nature, because none of them was remotely stable. We've already seen that the speed of rotation has to be just right — slightly too slow and the system collapses, slightly too fast and it falls apart. Unfortunately the masses of the bodies also have to be perfectly identical, otherwise the symmetry doesn't hold. Likewise the positions in the circle have to be just right. If anything is even slightly wrong the system falls to pieces. It's like balancing a ping-pong ball on a needle, theoretically possible but practically impossible.

Similarly, the counter-Earth orbit in Figure 5 is unstable. The two smaller masses have to be exactly identical, and travelling at exactly the right speed, otherwise the system falls rapidly apart. All of Euler's solutions are unstable. However, within a few years another mathematician, the Frenchman Joseph Louis Lagrange, came up with a new, stable, set of solutions to Newton's equation of gravity.

Lagrangian Nodes

Lagrange spent much of his life investigating the stability of the solar system, carefully calculating the effects of each of the planets on each other to see if there might be any circumstances under which the apparent stability of the solar system might be disrupted. This is still an active area of research, but, don't be worried, the solar system won't fall apart any time soon.   Lagrange was keen to investigate the circumstances under which a small object such as an asteroid or Moon could co-exist stably with an existing two-body system consisting of a larger body, such as Jupiter, and a very large body, such as the Sun.   The Moon and the Earth would be another suitable pair.

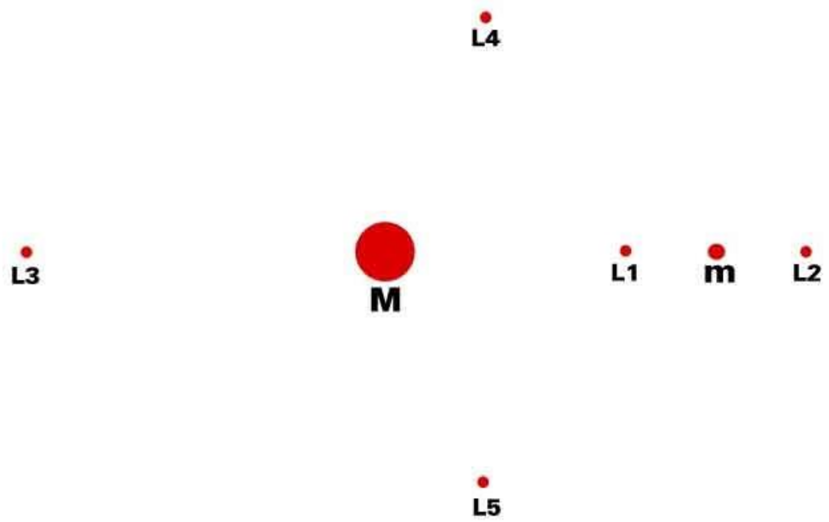


Figure 8 — Lagrangian Nodes. M is a very large mass for example, the sun, m is a medium mass, for example Jupiter. The Lagrangian Nodes L1 to L5 are points where a small mass will orbit without being pulled away by M and m. However only L4 and L5 are stable.

Lagrange's analysis followed Euler's insight in investigating the symmetries of possible systems. I won't bog you down with the detail (Vorobyov's article has a proof). Lagrange found five locations where a small body could be placed relative to two larger ones, so that their combined pull acted to keep the small body in configuration. Figure 8 indicates the five locations, known as the Lagrangian Nodes (see also Ivor Clarke's excellent editorial in MIRA 66). The first three locations are unstable — a small body placed there will, eventually, be pulled away by perturbations. Nevertheless, these Lagrangian Nodes are of practical use to space agencies, who park space probes there because it minimises the amount of fuel required to keep the probes in place. The SOHO solar observatory sits at the Sun-Earth L1 point, where it has an unobstructed view of the Sun, and the Wilkinson Microwave Anisotropy Probe WMAP observes microwave background radiation from the Sun-Earth L2 point, well clear of the Earth's glare. The L3 point corresponds to the counter-Earth position.

But what of the stable Lagrangian Nodes L4 and L5? These complete the equilateral triangle either side of the two larger masses. Any small mass placed at this location will be held there by the gravitational pull of the other two elements in the triangle. You might think that the configuration, an equilateral triangle, is exactly the same as Euler's unstable system. But there are hugely important differences. The three masses in the system are not identical but rather radically different, and the net effect is to place the centre of gravity not at the centre of the triangle, but very closest to the most massive of the three bodies. The triangle, to a good approximation, rotates around the largest of the three bodies.

Lagrange published this analysis in 1772, but it was not until 1906 that a natural occurrence of the phenomenon was discovered. Max Wolf discovered an asteroid, Achilles, at an unusually large distance from the Sun, and it soon became clear that Achilles was at the L4 point of the Jupiter-Sun system. The discovery of Achilles was followed by a series of other asteroid discoveries in the L4 and L5. Following the initial naming of Achilles, those at the L4 point were named after Greek heroes from the Trojan war, those at L5 after Trojan soldiers, and the generic name of "Trojan asteroids" given to all asteroids which are trapped at Lagrangian nodes. The largest Trojan asteroid, Patroclus, is 216 km across. As we shall see, the smaller Trojans can wander a surprising distance from the L4 and L5 points during their orbits round the Sun.

At a recent count, the Sun-Jupiter system kept a retinue of 1600 Trojan asteroids. The Sun-Mars pairing had seven Trojans and Sun-Neptune one Trojan. There are also Trojan satellites associated with Saturnian moons Dione and Tethys. Finally, in 1959 the polish astronomer Pan Kordylewsky noted large, very fine clouds of dust at the Earth-Moon Lagrangian nodes, so in a sense Earth itself maintains a retinue of tiny Trojan satellites. Isaac Asimov once proposed the Lagrangian Nodes of the Earth-Moon system as a safe dumping ground for nuclear waste — although transporting waste there might not be such a safe exercise!

Tadpoles and Horseshoes

There is actually a whole class of orbits taken by "orbit-sharing" asteroids, of which "stationary" location at the Lagrangian Nodes is but the simplest solution. Excursions from the Lagrangian Nodes, viewed from a rotating frame of reference, look like "tadpole" or "horseshoe" orbits. Figure 9 attempts to sketch such orbits. Note that the orbits are not "really" horseshoe-shaped, with the implied sudden changes of direction; likewise Jupiter doesn't stay still. Viewed by a non-rotating observer, both Jupiter and a Trojan asteroid appear to rotate around the Sun in near-elliptical orbits. The asteroid, however, will slowly migrate towards and away from Jupiter along the horseshoe path over a period of many orbits and hundreds of years.

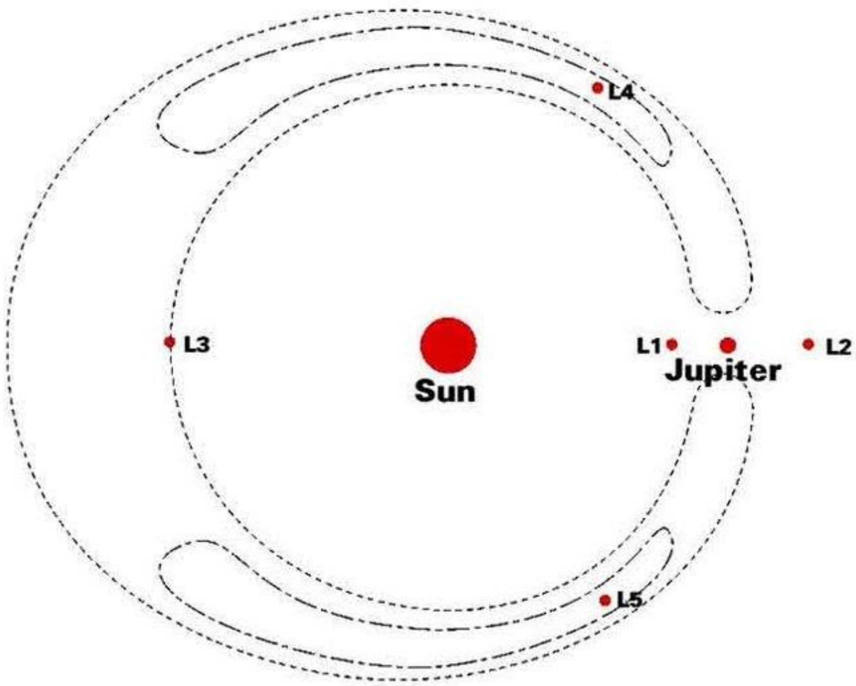


Figure 9 — Tadpole and Horseshoe orbits

Horseshoe orbits can be explained in terms of Kepler's third law, which we haven't so far discussed. Forgetting the mathematics, the third law tells us that the closer an object is to the Sun, the quicker it takes to complete an orbit. Mercury, for example, orbits the Sun in 88 days, Venus 243 days, Earth 365 days. Consider the case of Jupiter and a Trojan asteroid that is on the inner part of its horseshoe orbit. To a good approximation, the Trojan and Jupiter follow two elliptical orbits, one just inside the other. Because of Kepler's third law, the Trojan asteroid gradually catches up Jupiter over the course of many orbits. When it gets close enough to Jupiter to fall into its zone of attraction, the effect of Jupiter's gravity is to shunt it further away from the Sun than Jupiter. The Trojan moves away from Jupiter and back into the Sun's zone of influence. It is now on an elliptical orbit slightly further from the Sun than Jupiter, and therefore falls further and further behind Jupiter — a state of affairs that prevails until Jupiter catches it up from behind and swings it back into the an orbit closer to the Sun. The horseshoe therefore consists of long periods spent just inside and just outside the orbit of Jupiter, punctuated by sudden changes of orbit as it strays too close to Jupiter.

It was to be another 74 years before the next new type of orbit was found — and this time the discovery came from observation rather than theory.

Orbit Sharing

In 1980, Voyager 1 reached Saturn. One of the many tasks that the mission scientists hoped to accomplish was to recover Saturn's innermost moon, Janus. Janus had been discovered from Earth in 1966, although as it was so close to the top of the atmosphere of Saturn, it had proved very difficult indeed to observe against the much brighter disk of the planet. The Voyager team certainly found Janus, but they found more besides. There was a second moon, named Epimetheus, sharing almost the exact same orbit. And when the orbits were analysed further the scientists were surprised. The two satellites regularly swapped position!

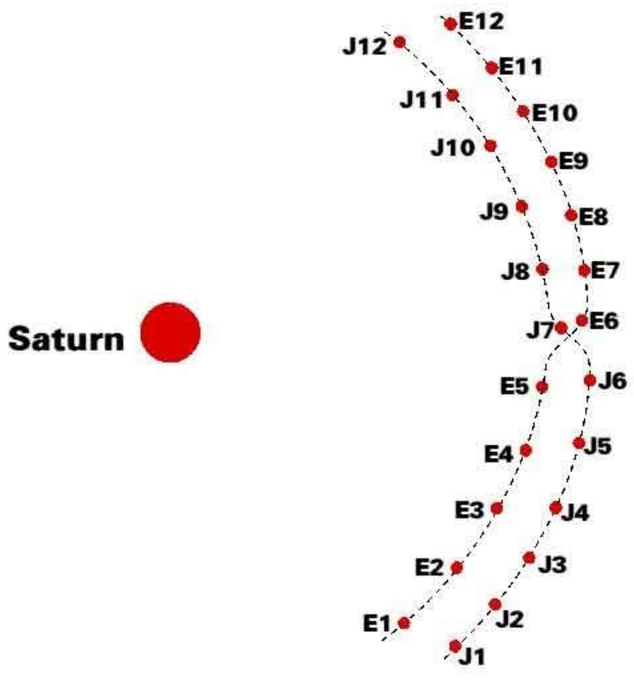


Figure 10 — Orbit swapping by Janus and Epimetheus

The explanation turned out to be similar to that for horseshoe orbits, except that instead of featuring two objects of radically different size - Jupiter and a Trojan asteroid - Janus and Epimetheus were much closer in size to each other. What happens is this. Most of the time, Janus and Epimetheus follow two elliptical orbits, one just inside the other. Let's say Janus is the innermost moon. For almost all its orbit the nearest "big thing" is Saturn, and so Janus orbits Saturn according to Kepler's laws, as does Epimetheus. Kepler's third law tells us that Janus will orbit Saturn rather faster than Epimetheus does. So, over the course of many orbits of Saturn, a period of four years, Janus will gradually catch up Epimetheus. Because the orbits are so close to each other, when Janus catches up Epimetheus the mutual attraction of the two moons will, for a short period, be greater than that of Saturn on either. The pull of Epimetheus drags Janus into the outer orbit and the pull of Janus drags Epimetheus into the inner orbit. Epimetheus goes into the inner track, Janus to the outer track, and the cycle begins once again with Epimetheus doing the catching-up this time. See Figure 10.

Nobody had predicted this behaviour. Not because it is a radically new piece of physics, but simply because nobody had thought to look for it - either theoretically, in simulations, or in nature. It is interesting to speculate how nature found a solution that had eluded the mathematicians. The most likely cause for the shared orbit is that Janus and Epimetheus were once both part of the same body, which was shattered in a collision with another object.

Perhaps the realisation that there were new solutions to Newton's equations, just waiting to be found, caused a resurgence of interest in solutions to the 3-body problem.

Gravitational Choreography

The next advance was made by running numerical simulations. In 1993, Cris Moore, an American mathematician, found a completely new solution that involved three identical masses. Euler, you will recall, found a solution where the three masses orbited in a circle, also a solution where the three objects rotated, staying in a straight line. Moore found a solution (Figure 11, below) where three masses orbited along a figure-of-eight.

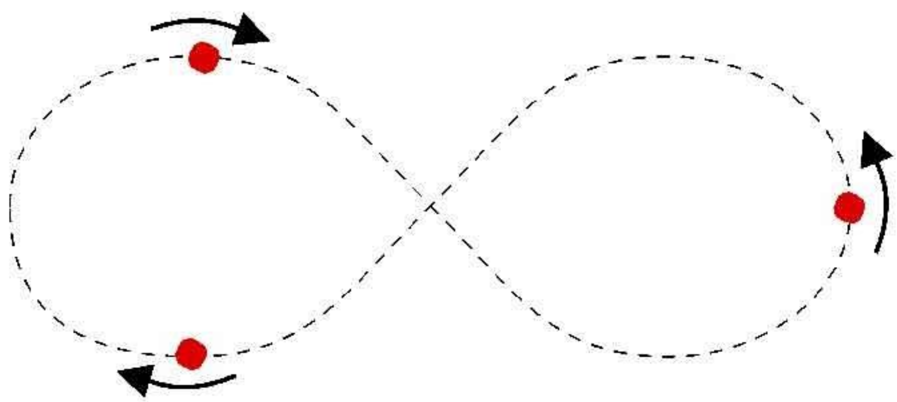


Figure 11 — The Moore - Chenciner Figure of 8 orbit. Masses move slower on the "outside bends" of the orbit and faster through the cross over points.

I find the Moore solution fascinating.   The three bodies follow each other endlessly round the figure of eight. Each body in turn speeds up as it flies through the crossover, then slows down at the outermost reaches of the eight, waiting its turn to pass through the crossover again. *Astronomy Now* described the orbit as *"an eightsome reel"*, after the Scottish country-dance, and indeed this and the solutions I'm about to describe are now categorised as gravitational "choreographies" as they mimic some of the features of traditional dances such as reels [6].

Moore's solution was radical but received surprisingly little attention. Interest exploded seven years later, when Alain Chenciner of France and Richard Montgomery, another American, rediscovered the Moore figure-8 orbit, and many others besides, from a theoretical point of view. They returned to the symmetry arguments that Euler and Lagrange had used centuries earlier, and, with the assistance of modern numerical techniques, developed them out of all recognition.

The results are simply stunning. I have included a selection of diagrams (Figure 12), taken from one of the papers I used to research this article (Glendinning, 2002), but I urge you — implore you! — to visit the web site run by Charlie McDowell. This is the URL:

http://www.soe.ucsc.edu/~charlie/3body/

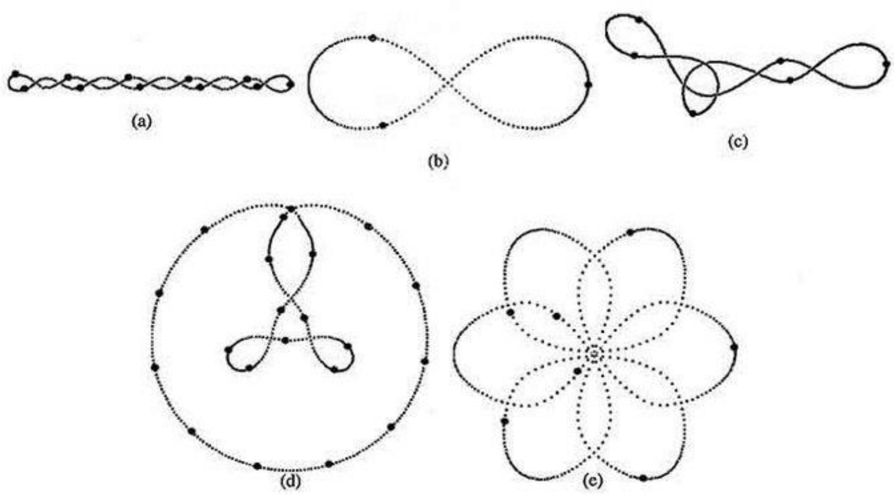


Figure 12 — 5 Multi-body choreographies, with names taken from a website. The particles move around the dotted paths, each particles position at an instance in time is indicted by a larger dot, which has been added to the original diagram. (a) 11 bodies on a 10 chain. (b) The Moore-Chenciner-Montgomery choreography: three bodies on a figure of eight. (c) Six bodies, non-symmetric. (d) Twenty-one bodies on a ?? (sic).

(e) Seven bodies on a flower.

The solutions are all for identical bodies of the same mass. Watching seven bodies tracing out the petals of a flower, or eleven bodies "stripping the willow" on a chain, is mesmerising. Some of the shapes depicted defy easy description.   But they are all legitimate N-body solutions to Newton's theory of gravity.

As you might guess, however, they are unstable solutions. As with Euler's solutions to Newton's equation, the slightest mismatch in masses, positions or speeds will cause the choreographies to fall to pieces. So it is practically impossible for such choreographies to arise naturally. Glendinning suggests practical reasons why they might be set up, for example the storage of large masses in space, but my own feeling is that the only sensible reasons for creating these choreographies in reality is aesthetic. Certainly if space explorers ever unexpectedly came across a choreography, it would be powerful evidence for the existence of an unknown civilisation. Potential gravitational choreographers would have to ensure that their building blocks were not only identical, but also sufficiently large for disturbances like radiation pressure to be negligible, whilst not too large or too dense that relativistic corrections have to be made to the theory of gravity. Asteroid size masses would be ideal.

However: it isn't quite true to say that all the new solutions are unstable. The figure of eight solution, discovered numerically by Cris Moore and theoretically by Chenciner and Montgomery, is stable — just. Provided that the three masses are the same, to within an error of 2%, the figure of eight orbit will survive small perturbations. Even if the initial configuration isn't spot on, the figure of eight orbit will prevail. The 2% mass tolerance is a pretty tight constraint; additionally, it isn't clear how such an orbit would form in the first place. But there is a scientific maxim, *"that which isn't forbidden will exist".* So the indications are that somewhere in this astonishing universe of ours — maybe not in this galaxy, maybe in the next — three stars, or planets, or moons, of nearly identical mass, will be dancing an eternal Scottish reel on a figure of eight orbit.

And I like to think that there's a UFO parked nearby. Isaac Newton sits at the window, munching an apple, puma purring contentedly on his lap, and he's thinking to himself, *"my theory predicted all this".*

And then he smiles.

Notes:

1] I also gave the talk in Wollongong, New South Wales, although I travelled to Australia round the outside of the Earth rather than through the middle.

2] You may have heard this anecdote before, but I repeat it for completeness. *"Standing on the shoulders of giants"* (Not "the shoulder" as Oasis mis-quoted it on an album cover in 2000) comes from a longer saying by Isaac Newton, *"If I have seen further than others, it is because I have stood on the shoulders of giants"*. It has been suggested that he was actually implying that he had taken nothing from his fierce rival, the diminutive Robert Hooke. Actually Newton borrowed the saying from earlier sources, notably Bernard of Cluny.

3] At least, it was the A-Level syllabus when I was doing it! I hope that "grade-slide" hasn't degraded the content as much as some people suggest.

4] It's possible to think of the slingshot effect as a local consequence of Kepler's second law. Consider Voyager, say, en route to Jupiter, in an elliptical orbit around the Sun. When Voyager entered the zone of attraction of Jupiter, from Jupiter's point of view it was approaching the planet very quickly on a hyperbolic orbit. Voyager swung rapidly around Jupiter and left on the other branch of the hypbola, never to return. When Voyager left Jupiter's zone of attraction it was once again in an orbit round the Sun, only one in which it was now traveling far more quickly.

5] There is one 3-body system within our solar system that doesn't fit comfortably into the "zone of attraction" model I put forward. The 3 bodies are the Sun, the Earth and the Moon, and the problem is that the Sun attracts the Moon more strongly than the Earth does. So why does the Moon continue to orbit the Earth? One of these days I will understand exactly how the Earth/Moon system works. Then I'll tell you about it.

6] Veteran members of the society may also remember another (short) talk that I gave, in about 1986, called *"Galactic square dances"*, in which I talked about stellar orbits around the galaxy. This isn't the same thing as the orbits I have described in this essay ! Stellar orbits are solutions to the "billion-body problem" where stars move in the gravitational field generated by all the rest of the stars in the galaxy. The orbits generated in this case are chaotic but bear resemblances to curves generated by spirographs.

Sources/Further Reading:

Isaac Asimov, *"Asimov on Astronomy"*, Ch 4. "*The Trojan Hearse"* and Chapter 10. *"Just Mooning Around"* (Coronet, 1976)

Ivor Clarke, MIRA 66, P2.

Paul Glendinning, *"View from the Pennines: Newtonian Choreography"*, Mathematics Today, December 2002.

Brindsley le Poer Trench *"Secret of the Ages — UFOs from inside the Earth"* (Souvenir Press, 1974)

I. Vorobyov, *"When Trojans and Greeks Collide"*, Quantum Magazine, Sep-Oct 1999

Web Pages

`http://www.soe.ucsc.edu/~charlie/3body/` New Choreographies

`http://www.arachnophile.com/gravitation` Conventional N-body simulations

VISION OR VANDALISM?

By Philip Spratley

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This is a copy of an article sent to Musical Opinion and to the Guild of Church Musicians for publication in the magazine Laudate. While it is not the usual type of subject for MIRA, I suspect most members have enjoyed listening to Holst's Planets Suite at some time, so what do you think of an extra section added to this famous work?

There are a number of musical works which have nothing to do with church music yet are very 'spiritual' in content and atmosphere. The Symphony by Cesar Franck immediately comes to mind as does, of course, the final movement of the sixth symphony of Vaughan Williams where the indecision of the strings to end in E flat or E minor gives a feeling of absolute nothingness. My guess is that the top of the list would be Holst's *'Planets Suite'*, music which goes into uncharted territory and which leads the mind to journey into vast new unlimited areas.

There was a performance of Hoist's master-piece on March 20th on BBC2 which was visually very interesting and arresting and the BBC must be applauded for this experiment. After 'Neptune the Mystic' there appeared a caption about the discovery of Pluto in 1930 and the subsequent composition by Colin Matthews of a piece in 2000 of the same name 'to complete the suite'. This suggestion is really astonishing and diabolical.

Is not 'The Planets' already complete and, when the female choir dissolves into the remote regions of outer space, does there not end one of the most striking and original works of any age? Had it been written by a continental composer, it would have reached the parts where British music unjustly fails to reach. By virtue of its length and character it usually is the last work in a concert programme. Incomplete performances of the work distressed Holst and when American concerts were arranged he told his agent that it should be a case of 'all seven movements or none'. What on earth would he say about this monumental piece of gate-crashing? After the discovery of Pluto he had over four years to consider composition on this subject but, understandably, felt his work was complete.

Most music lovers understand that the inspiration behind the 'Planets' is astrology, and NOT Astronomy. I therefore looked to see what the sub-title of the new piece was. Would it be Pluto the Presumptuous? Pluto the Polydissonant? Pluto the bringer of Pessimism? There did not seem to be anything to inform us. It was necessary to visit the composer's web-site and find that it was to be Pluto the 'renewer', whatever that means. I followed this by reading the sleeve notes for one of the C.D. recordings.

It is here that the whole situation gets to be totally absurd and out of hand. The composer has already experienced some flak about this work and has tried to justify himself by convincing his listeners that he understood the Astronomical facts of the outer solar system before he began work on his piece. Since he and others are unable to separate the two studies of the universe the following information might be useful. In 1992 the astronomers David Jewitt and Jane Luu discovered yet another remote object in the Solar System. This was the first discovery of what are known as Kuiper Belt Objects (KBO's), debris left over from the formation of the Solar System. Most lie further from Neptune than the distance of Neptune from the Sun. Some 900 are known but are NOT regarded by astronomers as planets. The record holder for size was an object dubbed 'Quaoar' (pronounced KWA-WAR and named after a native American spirit) discovered in 2002 but now superseded. In March of this year (2004) another of significance has been found; this has been given the name Sedna and in size it is comparable with Pluto! Before composers rush for a pen to beat Matthews the chord of the supertonic 19th must be sounded.

There is controversy amongst astronomers as to the status of Pluto as a planet. It is argued by many, including Prof. Jewitt that it is Pluto, not Quaoar that is really the first KBO to be discovered. Heather Couper, the well known astronomer and TV personality agrees that Pluto and its companion, Charon, both smaller than our own Moon, should be demoted to KBO's. If this occurs, then it is obvious what should happen to any music called 'Pluto', 'Quaoar', 'Sedna', KBO, Old Uncle Tom Asteroid and all.

Everyone who loves English music should be grateful to Colin Matthews for his work with Imogen Holst in the resurrection of some of Holst's long forgotten music. But why could he not be satisfied with that? He says he did not attempt to impersonate Holst. What he has done is to appropriate Hoist. The situation is even more absurd when he admits to being a sceptic as far as astrology is concerned. Where then is the inspiration for the piece? In these material times of ours are commercial considerations more important than people's feelings? Is it a case of someone desperately short of publicity? The final insult is the fact that the piece has been dedicated to the memory of Imogen Hoist. She, I guess, would not be amused at all. Dismayed, yes, and even betrayed.